$$BM:\quad l\alpha + n\gamma = 0, \qquad BQ:\quad p\alpha + r\gamma = 0,$$

$$CN:\quad l\alpha + m\beta = 0, \qquad CR:\quad p\alpha + q\beta = 0,$$

$$RM:\quad lp\alpha + lq\beta + np\gamma = 0,$$

$$QN:\quad lp\alpha + mp\beta + lr\gamma = 0.$$

At the intersections of BM and CN, of BQ and CR, and of RM and QN, the coördinate ratios are respectively:

$$\begin{aligned} \alpha_1 : \beta_1 : \gamma_1 &:: \quad -mn \quad : \quad nl \quad : \quad lm, \\ \alpha_2 : \beta_2 : \gamma_2 &:: \quad -qr \quad : \quad rp \quad : \quad pq, \\ \alpha_3 : \beta_3 : \gamma_3 &:: l^2rq - p^2mn : p^2nl - l^2rp : p^2lm - l^2pq. \end{aligned}$$

The vanishing of the determinant of these three sets of coördinates proves the proposition.

Also solved by NATHAN ALTSHILLER, H. C. FEEMSTER, and F. M. MORGAN.

444. Proposed by S. A. COREY, Hiteman, Iowa.

Let $ABCDE$ be a pentagon, plane or gauche, with sides AB, BC, CD, DE, and EA. Bisect BC and DE in H and K respectively. Extend AB from B to B', and AE from E to E'. On AB' take sects AP and AV, and on AE' take sects AL and AT. Draw AD, AC, AH, AK, and DT. Let a, b, c, and d equal AL/AE, AT/AE, AV/AB, and AP/AB, respectively. Extend (or contract) AC from C to W, and AD from D to S, making $AW = a \times AC$ and $AS = d \times AD$. Draw LM and PN parallel to, and of the same currency as, AD and AC respectively, and of lengths $c \times AD$ and $b \times AC$, respectively. Draw AM, AN, ST, and WV. Draw DQ and VX parallel to, and of the same currency as, CB and TS, respectively. We are to prove that $2(ad + bc)(AK \times AH \times \cos KAH + KE \times HC \times \cos QDK) = AM \times AN \times \cos MAN + TS \times VW \times \cos WVX$.

SOLUTION BY THE PROPOSER.

Let KE, HB, AH and AK, in the proposed figure, be represented by the vectors x, y, z, and w, respectively. Then by vector addition, $w + x = AE$, $w - x = AD$, $z + y = AB$, $z - y = AC$; by construction, $\angle ST \cdot WV = \angle WVX$, $\angle KE \cdot HB = \angle QDK$, $(w + x)a = AL$, $(w - x)c = LM$, $(z + y)d = AP$, $(z - y)b = PN$, $(w + x)b = AT$, $(w - x)d = AS$, $(z + y)z = AV$, and $(z - y)a = AW$; also by vector addition, $(w + x)a + (w - x)c = AM$, $(z + y)d + (z - y)b = AN$, $(w + x)b - (w - x)d = ST$, and $(z + y)c - (z - y)a = WV$.

Consider now the algebraic identity,

$$\begin{aligned} &(w + x)a + (w - x)c][(z + y)d + (z - y)b] \\ &\quad + [(w + x)b - (w - x)d][(z + y)c - (z - y)a] = 2(ad + bc)(wz + xy) \end{aligned} \tag{1}$$

and note that when fully expanded each term is of the second degree in x, y, z, and w. Also note that the identity may be written

$$AM \times AN + ST \times WV = 2(ad + bc)(AK \times AH + KE \times HB) \tag{2}$$

if we substitute vectors as above indicated.

Inasmuch as vector multiplication is commutative if no term of the product is of a degree higher than the second in the vectors employed, and if the scalar part only of the resulting product be considered, we may assume that the algebraic identity (1) has a geometric interpretation which may be derived from (2) by considering the scalar part only of the vector products indicated in (2). The scalar part of the product of two vectors may be taken as the positive product of the lengths of the vectors into the cosine of their included angle. Placing this interpretation on the scalar part of the products indicated in (2) the equation of the problem is at once obtained, and the truth of the theorem established.

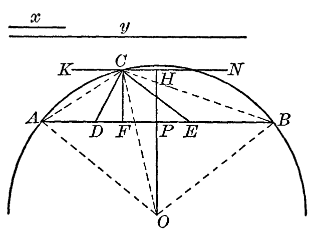
As no assumption has been made restricting any of the lines to any one plane the pentagon may, of course, be either plane or gauche. To help form a mental picture of a gauche pentagon, let $ABCD$ be a tetrahedron, with edges AB, BC, CD, DA, AC, and BD, and let E be a point within or without such tetrahedonr. Then if E be connected with A and D by right lines the figure $ABCDE$ will be a gauche pentagon.

445. Proposed by CLIFFORD N. MILLS, South Dakota State College.

Given the perimeter of a right triangle and the perpendicular falling from the right angle on the hypotenuse, to determine the sides of the triangle.

SOLUTION BY G. I. HOPKINS, Manchester (N. H.) High School.

Let x be the altitude and y the perimeter. Draw $AB = y$, and OH its $\perp$ bisector. Make $PH = x$, and draw KN through $H \perp$ to HO. Make $PO = AP$. With O as center and radius OA describe the circle ACB. Draw the chords CA and CB, and the radii OA, OB, and OC. Make $\angle ACD = \angle CAD$, and $\angle BCE = \angle CBE$. $\therefore DCE$ is the $\triangle$ required. For,



$$\angle AOP = 45° = \angle POB. \quad \therefore \angle AOB = 90°.$$

$$\angle OAC = \angle OCA \text{ and } \angle CAD = \angle ACD. \quad \therefore \angle OCD = \angle OAD = 45°.$$

In like manner $\angle OCE = 45°$; $\therefore \angle DCE = 90°$, $AD = DC$, and $EB = CE$; $\therefore$ the perimeter of the $\triangle DCE = y$, CF is $\perp$ to AB, $\therefore CF = HP = x$.

Note. The figure is not accurate as OP is not made equal to AP.

Also solved by B. J. BROWN, A. H. HOLMES, C. N. SCHMALL, and NATHAN ALTSHILLER.

CALCULUS.

356. Proposed by F. B. FINKEL, Drury College.

A steel girder l ft. long and w ft. wide is moved along a passageway a ft. wide and into a corridor at right angles to the passageway. How wide must the corridor be to admit the girder?